# TOPOLOGY

HW4: SELECTED HINTS

1. a. The set of $\mathbb{Z}$ in $\mathbb{R}$: Not Compact. Consider $(-n/2, n/2) \cap \mathbb{Z}$

   b. { 1/n : n $\in \mathbb{N}$ }: Not compact since not closed.

   c. {(x,y): y = $\cos x$, y $\in$ [0,1]} : Closed and Bounded in euclidean plane therefore compact.

   d. {(x,y): y=$\tan x$, y $\in [0, \pi/2]$} : Not compact since unbounded at $\pi$ /2

2. (0,1) is not compact.

   Consider {(1/n,1): n $\in \mathbb{N}$} as the open cover.

3. $[0, 1]$ is not homeo to (0,1) Suppose f:[0,1] -> (0,1) a homeo. [0,1] is compact and (0,1) is not.

   Contradiction.

4. $\mathbb{Q} \cap [0, 1]$ is not compact since it is not closed in [0,1]

5. Two compact sets A & B in Hausdorff space have disjoint open nbds.

   Solution : Fix a $\in$ A. $\forall$ b $\in$ B find disjoint neighborhoods for a and b. Then the open sets for b form an open cover of B which admits a finite subcover. Consider the intersection of open sets around a corresponding to the finite subcover of B.

   We have $U_a$ around a and $\cup V_b$ around B. Now $\forall$ a $\in$ A consider similar nbd. A is compact therefore it has a finite subcover. Consider that finite subcover as an open set U around A. Take the intersection of $V_b$ corresponding to the finite subcover of A giving us V around B. Therefore U and V are disjoint open sets around A and B respectively.

6. A is compact subset of a metric space X. Given x $\in$ X, show d(x,A) = d(x,y) for some y $\in$ A.

   d(A,B) = inf{d(a,b): a $\in$ A, b $\in$ B} If B is closed and disjoint from A, then d(A,B) >0.

   Solution : Consider the function f: A $\to$ $\mathbb{R}$ defined as f(y) = d(x,y). Since A is compact the function achieves the bound. Therefore $\exists y_0$ such that d(x,A) = d(x,$y_0$) Similarly for function g: A $\to$ $\mathbb{R}$ defined as g(x) = d(x,B). It attains the lower bound and therefore $\exists$ $x_0$ such that d($x_0$,B) = d(A,B). But $x_0 \notin$ B and B closed $\Rightarrow$ d(A,B) >0.